Statistics
Winter 2022
Lecture 12

Live QZ Extra Credit

Consider a binomial Prob. dist with $n=400$, and $P=.5$

1) P(exactly 195 Successes)

$= P(x=195) = \text{binompdf}(400, .5, 195) = \boxed{.035}$ ✓

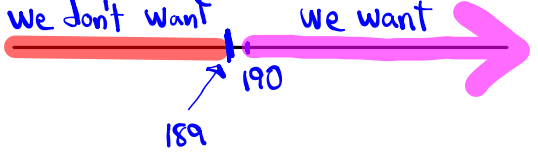
2) P(at most 210 Successes)

$= P(x \leq 210) = \text{binomcdf}(400, .5, 210) = \boxed{.853}$ ✓✓

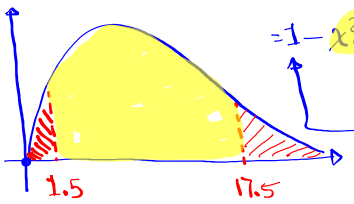
3) P(at least 190 Successes) $= \boxed{.853}$ ✓✓✓

$= P(x \geq 190) = 1 - P(x \leq 189) = 1 - \text{binomcdf}(400, .5, 189)$

Total Prob.



Find $P(\chi^2 < 1.5 \text{ OR } \chi^2 > 17.5)$ with df = 10



$= 1 - \chi^2cdf(1.5, 17.5, 10)$

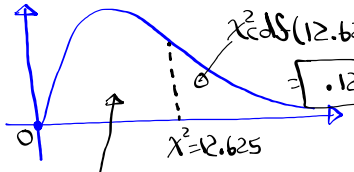
L U df

Total Area = 1

Total Prob. = 1

$= .065$

Find the area on each side of $\chi^2 = 12.625$ with df = 8, then multiply the smaller area by 2.



$\chi^2cdf(12.625, E99, 8)$

$= .125$

$\chi^2cdf(0, 12.625, 8)$

$= .875$

Total Area = .875 + .125 = 1

2 * Smaller area = 2(.125)

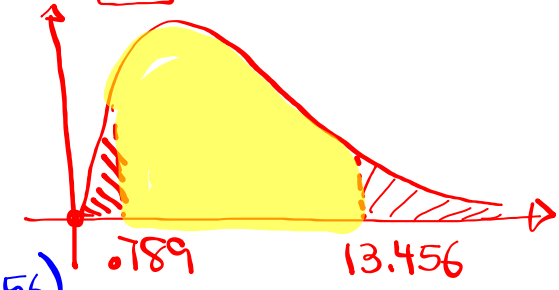
$= .250$

Find $P(F < .789 \text{ and } F > 13.456)$ with

Ndf = 5 & Ddf = 25.

0

M.E.E.

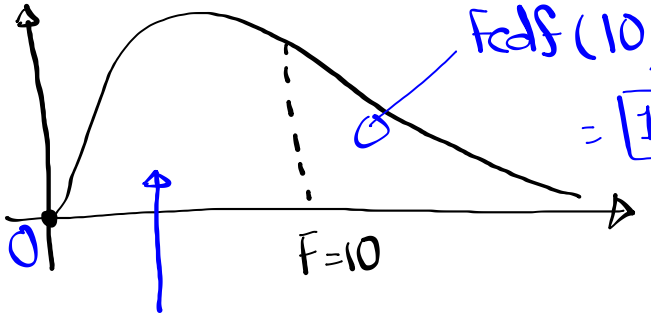


$P(F < .789 \text{ OR } F > 13.456)$

$= 1 - P(.789 < F < 13.456) = 1 - Fcdf(.789, 13.4565, 25)$

$= .432$

Find the area on each side of $F = 10$ with $Ndf = 4$ & $Ddf = 20$, then multiply the smaller area by 2.



$fcdf(10, E99, 4, 20) = 1.3 \times 10^{-4}$

$fcdf(0, 10, 4, 20) = .99987 \approx 1$

2 * Smaller area $= 2(1.3 \times 10^{-4}) = 2.6 \times 10^{-4}$

What is degrees of freedom?

It is # of choices.

It will be computed differently for different topics.

ex: 20 students in the class.

I bring 20 Donuts.

First student $\Rightarrow$ 20 choices

2nd " $\Rightarrow$ 19 " $\quad df = 19$

Third " $\Rightarrow$ 18 "

⋮ ⋮ ⋮

Last " $\Rightarrow$ 0 choice (1 Donut left)

Ex: Jose 7 clean shirts.

He wears one per day.

Monday → 7 choices

Tuesday → 6 "

Wednesday → 5 "

⋮

Saturday → 2 "

Sunday → (No choice) → 1 clean T-shirt left

$df = 7 - 1$

$df = 6$

SG ... - 22 ✓

SG 23 - 24

Estimate Parameters

Sample ↕ Statistic

Population ↕ Parameter

Population Proportion p ⇒ use Sample Proportion $\hat{p}$

Population Mean μ ⇒ use Sample Mean $\bar{x}$

Population Standard deviation σ ⇒ use Sample Standard deviation s

Point-estimate

Our final estimation of any parameter will be range of values (Confidence Interval)

$\_\_ < p < \_\_$, $\_\_ < \mu < \_\_$, $\_\_ < \sigma < \_\_$

Proportion | Mean | Standard Deviation

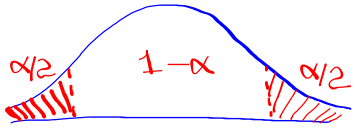
Every Confidence Interval comes with

Confidence level $\Rightarrow$ It is middle area of the graph of the Prob. dist.

$(1-\alpha)\cdot 100\%$

$\hookrightarrow$ Significance level $\rightarrow 0 < \alpha < 1$

$\alpha/2$ Right Tail, $\alpha/2$ left tail, $1-\alpha$ Middle area

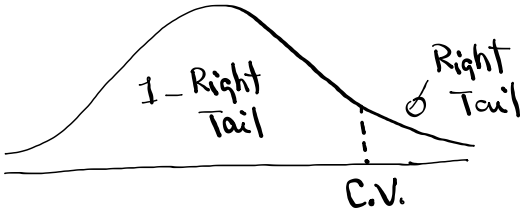


Popular Confidence level

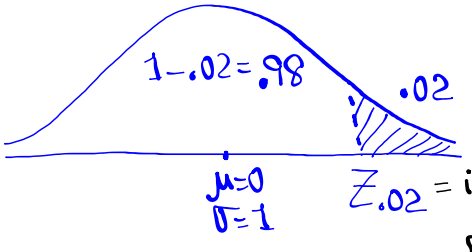
90%	$\Rightarrow$ Middle area = .9 ,	Right tail = .05 Left tail = .05
95%	$\Rightarrow$ Middle area = .95	Right Tail = .025 Left Tail = .025
98%	$\Rightarrow$ Middle area = .98	Right-Tail = .01 Left-Tail = .01
99%	$\Rightarrow$ Middle area = .99	Right-Tail = .005 Left-Tail = .005

When C-level is not given, use 95% C-level.

The value that separates the right tail from the rest is called **Critical Value**.



ex: Find $z_{.02}$ — Right Tail Area



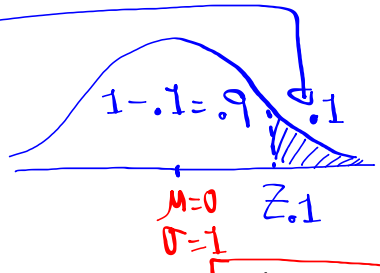
$z_{.02}$ = invNorm(.98, 0, 1)

= **2.054**

Find $z_{.1}$

= invNorm(.9, 0, 1)

= **1.282** ✓✓

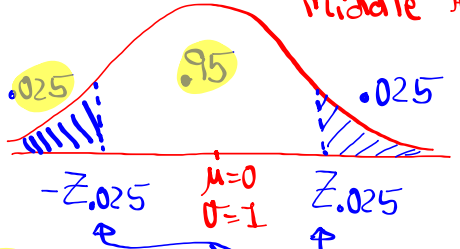


Find two Critical Values z for **95% C-level.**

Middle Area

1 − .95 = .05

.05 ÷ 2 = .025



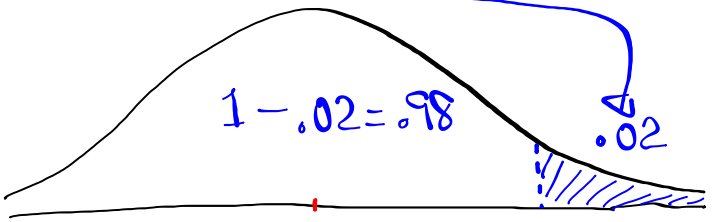
$z_{.025}$ = invNorm(.975, 0, 1)

= **1.960**

-1.960

1.960

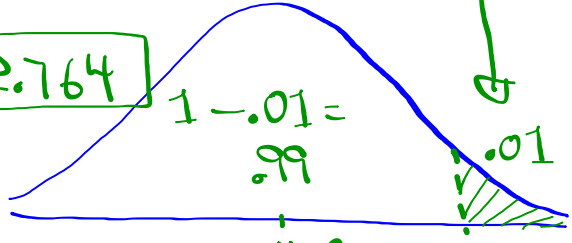
Find $t_{.02}$ with $df = 12$



Find $t_{.01}$ with $df = 10$.

invT(.99 , 10) = 2.764

left Area, df

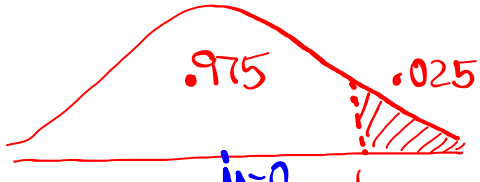


Find $t_{\alpha/2}$ with $\alpha = .05$ and $df = 19$.

$\alpha/2 = .05/2 = .025$

Find $t_{.025}$

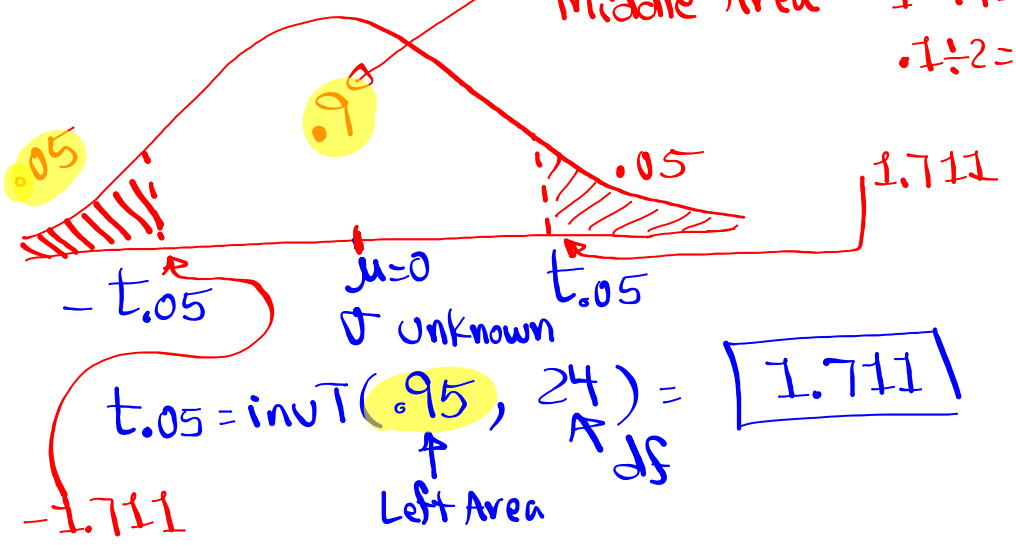
$t_{.025}$ = invT(.975, 19) = 2.093 ✓



Find $\pm t_{\alpha/2}$ for 90% C-level with df=24.

Middle Area $1-.9=.1$

$.1 \div 2 = .05$



σ unknown

$t_{.05} = \text{invT}(.95, 24) = 1.711$

(.95 ↑ Left Area; 24 ↑ df)

-1.711

Estimating Population Proportion p:

Final Ans $\quad < p <$

Format $\quad \hat{p} - E < p < \hat{p} + E$

$\hat{p}$: Point-estimate; E: Margin of error

Sample Proportion

$\hat{p} = \frac{x}{n}$

$\hat{q} = 1 - \hat{p}$

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$(1-\alpha) \cdot 100\%$ C-level

I surveyed 100 students, and 30 of them were a fan of Zoom lectures.

$\hat{P} = \frac{x}{n} = \frac{30}{100}$ $\hat{P} = .3$ $\hat{q} = .7$

Find 90% Confidence interval for the prop. of all students that are fan of Zoom lectures.

$\hat{P} - E < p < \hat{P} + E$

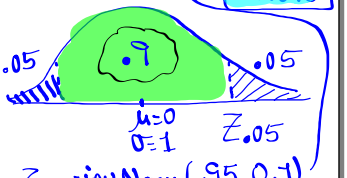
$.3 - .075 < p < .3 + .075$

$.225 < p < .375$

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\frac{(.3)(.7)}{100}}$

$E \approx .075$



$z_{.05} = \text{invNorm}(.95, 0, 1)$

STAT → TESTS ↓

1-PropZInt

x=30

n=100

C-level: .9

Calculate

$.225 < p < .375$

In a survey of 420 students, 65% of them were vaccinated.

$n = 420$

$\hat{P} = .65$

$\hat{q} = .35$

$\Rightarrow x = n\hat{P} = 420(.65) = 273$ always round-up

Find 98% Confidence interval for the prop of all students that are vaccinated.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\hat{q}}{n}}$

$= 2.326 \cdot \sqrt{\frac{(.65)(.35)}{420}}$

$= .054$

$\hat{P} - E < p < \hat{P} + E$

$.65 - .054 < p < .65 + .054$

$.596 < p < .704$

$z_{.01} = \text{invNorm}(.99, 0, 1)$

STAT → TESTS ↓

1-PropZInt

x = 273

n = 420

C-level: .98

Calculate

$.596 < p < .704$

$\hat{P} = \frac{.704 + .596}{2} = .65$

$E = \frac{.704 - .596}{2} = .054$ ✓✓

Among 575 LA residents, 62% of them were a fan of late Kobe.

$n = 575$

$\hat{p} = .62$

$\hat{q} = .38$

$\Rightarrow x = n\hat{p} = 575(.62) = 356.5$

$x = 357$

Find confidence interval for the prob. of all LA residents that are fan of Kobe.

NO C-level

$\Rightarrow$ use .95

$E = \frac{.661 - .581}{2} = .04$

$\hat{p} = \frac{.661 + .581}{2} = .621 \approx 62\%$

$.581 < p < .661$

1-PropZInt

$x = 357$

$n = 575$

C-level: .95

Calculate

Estimating Population Mean μ:

Final Ans: $\quad < \mu <$

Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Point-estimate

E: Margin of error

Case I: σ Known

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

ZInterval

I randomly selected 40 exams, the mean score was 85.

$n = 40$

$\bar{x} = 85$

Assume standard deviation of scores of all exams is 15.

$\sigma = 15$

Find 99% Confidence Interval for the mean of all exams.

C-level: .99

$78.891 < \mu < 91.109$

$79 < \mu < 91$

$E = \frac{91 - 79}{2} = 6$

$\bar{x} = \frac{91 + 79}{2} = 85$

σ Known $\Rightarrow$ Z Interval

STAT $\rightarrow$ TESTS $\downarrow$ ZInterval

inpt: STATS

$\sigma = 15$

$\bar{x} = 85$

$n = 40$

C-level: .99

I randomly selected 25 Mt. SAC students, their mean age was 30.8 yrs.

$n = 25$

$\bar{x} = 30.8$

$\sigma = 7.5$

Assume standard deviation of ages of all students is 7.5 yrs, find Confidence interval for the mean age of all students.

NO C-level $\Rightarrow$ .95

$27.86 < \mu < 33.74$

$27.9 < \mu < 33.7$

σ Known $\Rightarrow$ Z Interval

inpt: STATS

$E = \frac{33.7 - 27.9}{2} = 2.9$

$\bar{x} = \frac{33.7 + 27.9}{2} = 30.8$

Estimating Population Mean μ:

Final Ans: $\quad < \mu <$

Format: $\quad \bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Point-estimate

E: Margin of error

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\rightarrow df = n-1$
Z Interval	T Interval

Given: $n=16$, $\bar{x}=120$, $s=18$, C-level: .9

Find conf. interval for μ.

σ Unknown

$\Rightarrow$ T Interval

Inpt: (STATS)

$\bar{x}=120$ ← whole #

$s=18$

$n=16$

C-level: .9

Calculate

$112.11 < \mu < 127.89$

$112 < \mu < 128$

$E = \frac{128-112}{2} = 8$

$\bar{x} = \frac{128+112}{2} = 120$

12 randomly selected nurses had a mean salary of $6200 with standard deviation of $300.

$n=12$, $\bar{x}=6200$, $S=300$

Find confidence interval for the mean salary of all nurses.

$6009 < \mu < 6391$

No C-level $\Rightarrow .95$

σ Unknown $\Rightarrow$ T Interval

$$E = \frac{6391-6009}{2}$$

$E = 191$

Live QZ 4

Consider a binomial Prob. dist. with $n=275$ and $P=.6$

Find (Round to whole #)

1) $\mu = np$
$= 275(.6)$
$= 165$

2) $\sigma^2 = npq$
$= 275(.6)(.4)$
$= 66$

3) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{66}$
$\sigma \approx 8$